2-D Phase Unwrapping Algorithm Based on Pseudocorrelation Quality Map

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***Abstract*—Phase unwrapping is the mathematical problem-solving that appears in several research areas such as: Interferometric Synthetic Aperture Radar (InSAR), optical interferometry and adaptive optics. Phase unwrapping is an essential and critical step in interferometric processing. It consists in retrieving the absolute phase from modulo-2π phase. In this paper, we propose a 2-D phase unwrapping algorithm based on a contextual pseudocorrelation quality map and non-continuous path unwrapping. It measures the variability of the phase relative to the discontinuities and the transitions of the fringes in the interferogram. The proposed method has been tested using synthetic and real interferograms.**

***Index Terms*—InSAR, interferogram, phase unwrapping, quality map, contextual pseudocorrelation, unwrapping-path**

I. Introduction

The interferometric wrapped phase, calculated from the SLC (Single Look Complex) SAR images by the trigonometric arc tangent function, is uniformly distributed over the range [–π, π] [1], hence, the discontinuities modulo 2π appear in this phase distribution. The phase unwrapping is the process by which this discontinuities are resolved and the results are converted into continuous phase value [2]. The phase unwrapping problem has been discussed in several studies, looking for reliable solutions approaching the physical reality [3], [4], but a good solution is difficult to achieve. This is due to the complexity of the radar signal, the nature and the principle of interferometric phase, the geometry of the observed surface…etc. These perturbations are often modeled by a permanent noise in the wrapped phase causing abnormal values and perturbations in the phase unwrapping process. The principle of phase unwrapping is to find an estimate of the absolute phase $\emptyset$, from measured phase ψ. In many applications, $\emptyset$ is related to a certain physical quantities such as: surface topography in SAR interferometry [5]-[10], wavefront distortion in adaptive optics [11]-[13], etc. In this paper, we propose an algorithm for InSAR phase unwrapping based on the reliability of the unwrapping path. We considered a quality of phase measurement by the contextual pseudocorrelation, which follow a non-continuous unwrapping path. In order to validate our algorithm, the process developed has been tested on synthetic and real interferograms. We have generated our real interferogram from a pair of SAR SLC images acquired by ERS1/ERS2 on the 3rd and 4th January, 1996 of Algiers area images acquired by ERS1/ERS2 on the 3rd and 4th January, 1996 of Algiers area.

We will present in section II, the principle and the geometry of interferometry acquisition. In section III we will discuss the principle of phase unwrapping, and then we will describe the proposed algorithm. Finally we will conclude with a qualitative study of results.

II. SAR Interferometry

Interferometric synthetic aperture radar (InSAR) is used to produce a Digital Elevation Model (DEM) [14]. InSAR is one of many applications that use the technique of phase unwrapping [3]. It is based on the use of two complex images of the same scene, acquired from two different orbits separated by distance called 'baseline'. The acquisition geometry of interferometric pairs is shown in Fig. 1.

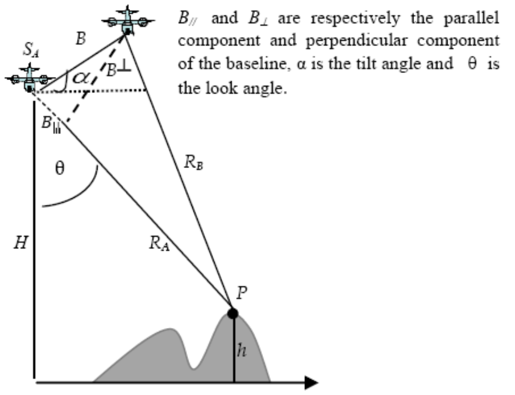

Figure 1. Geometric model of SAR interferometry.

Both sensors measure S_A and S_B signals backscattered by target P located at height h with distances R_A and R_B from two sensors A and B respectively.

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The two signals S_A and S_B are given by the following expressions:

$$S_A(P) = \delta_A e^{j\varphi_A(P)} \quad (1)$$

$$S_B(P) = \delta_B e^{j\varphi_B(P)} \quad (2)$$

where δ and φ are the amplitude and phase of the radar wave.

The phase $\varphi(P)$ of each signal is given by:

$$\varphi(P) = \varphi_{distance} + \varphi_{scatterer} + \varphi_{noise} \quad (3)$$

Phase includes three main terms:

- $\varphi_{distance}$: is the phase shift due to the round trip of the radar wave in vacuum.
- $\varphi_{scatterer}$: is the phase shift of the wave caused by its interaction with the target, it depends on the land reflectivity characteristics.
- φ_{noise} : is the phase shift due to the noise introduced by the instrument and acquisition process, the atmosphere and the geometry of the scene.

The interferometric complex multiplication is given by:

$$\chi(P) = \frac{\sum_F \left(S_A(P) \times S_B^*(P)\right)}{\sqrt{\sum_F |S_A(P)|^2 \times \sum_F |S_B(P)|^2}} = \gamma e^{j\psi} \quad (4)$$

where F is the averaging window.

$\psi = \varphi_A - \varphi_B$: is the argument or phase of the complex product. It is calculated for all pixels of the common scene of the two images. This image or interferogram contains fringes that represent phase differences that are in the range $[-\pi, \pi]$. These fringes can be split into two contributions: the first (topographic fringes) depends on the topographic of the scene (elevation h); the second is the orbital fringes or flat-earth phase which is due to the geometry of the satellites (baseline) [15]. This component must be subtracted from the global phase in order to restore the topographic component.

γ: is the correlation coefficient or coherence between the two complex signals (S_A and S_B). The coherence γ expresses a quality measure of the interferometric phase. Its value is near to1 if the ideal assumptions are met. In contrary, this term tends to 0 if the random noise contribution is important [16].

We present in Fig. 2a and Fig. 2b the interferogram and coherence of the east area of Algiers, (Algeria), respectively.

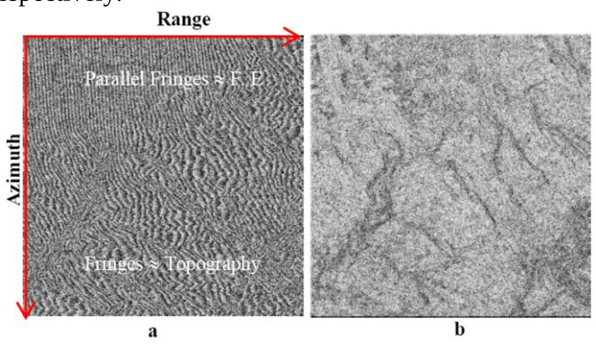


Figure 2. a: Interferogram, b: Coherence.

The dark parts of the coherence image reflect a decorrelation of the interferometric pair. On the interferogram, we can see mainly two regions: a hilly area, modeled by very narrow and tightened fringes (bottom of Fig. 2a), a relatively flat area, represented by large fringes dominated by orbital fringes according to their parallel structure with respect to the azimuthal axis. However, interferometric phase (topographic) is measured modulo 2π. In order to deduce the continuous phase, proportional to the topographic height of each target (pixel), a phase unwrapping step must be integrated in every interferometric process.

III. Phase Unwrapping

A phase unwrapping algorithm seeks to find an estimate of the absolute phase $\emptyset$ for each pixel in the interferogram. We aim to find the integer k, from the phase measurement ψ such that $\emptyset = \psi + 2k\pi$.

The Fig. 3 shows the scheme showing the relationship between ψ and $\emptyset$.

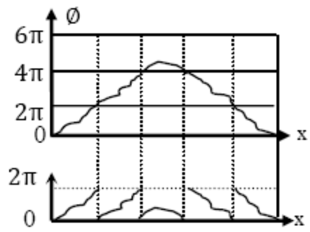


Figure 3. Principle of phase unwrapping.

Different methods have been proposed and developed to analyze, seek, model and /or estimate the absolute phase from the measured phase in noisy context [3], [17]-[19]. We can group them in three main approaches:

- The local approach: it is based on the progression following a continuous path from a reference pixel [20].
- The global approach: it seeks to minimize a global error function.
- The mixed approach (hybrid): in this case, the method is based on minimizing energy of interaction in an appropriate neighborhood [21].

The first approach is the basic principle of phase unwrapping. This is done by integrating phase difference of the pixels along the path. For each pair of pixels, the absolute value of the phase variation should not be greater than π. The phase jumps generate discontinuities due to noise [22]. However, these irregularities are quantified by residues. Remember that a residue, in normalized interferogram, is the algebraic sum of the pseudo- derivatives on a closed path. It has one of the following values $\{-1, 0, 1\}$ according to the absence or presence of discontinuities [20]. Several authors [3], [18], [19] tried to model these irregularities to adapt the local phase unwrapping approach. Among these methods, we find those that define the way of unwrapping by a quality of path-following which use the phase information, the coherence and/or the amplitude.

For this purpose, we propose an algorithm based on the selection and sequential location of a context with respect to a quality measure. The unwrapping process is then performed without error propagation.

IV. Proposed Algorithm

The main idea of the quality path-following unwrapping algorithms consists of the unwrapping of the highest-quality pixels first (Herraez *et al*, 1996; Xu et Cumming, 1996). The success or failure of these algorithms is strongly dependent on the quality map (Bone, 1991). Ref. [1] was the first who adopted a quality measure to guide the unwrapping process. He calculated the second-order partial derivative for each pixel in the context of 4-connectivity. With a decreasing threshold at each iteration, only pixels whose qualities exceed the threshold value are unwrapped first. The results of these algorithms depend on two basic elements: the reliability function used and the path of integration (unwrapping). In our algorithm, we use the pseudocorrelation resulted from two non-linearly dependent functions to calculate the quality map. But for the path of unwrapping, we introduced the notion of 8- incomplete-connectivity.

A. Quality Map Function

According to their data source, quality images (map) are classified into two categories: (i) the image quality derived from original data (coherence). (ii) The image quality derived from wrapping phase (interferogram) [22]. Quality maps of the first category are an indicator of the quality of the interferometric product. They are used indirectly to define the path of unwrapping. Concerning the second category, it shows the transition variability, within fringes. They are deduced from interferogram.

The pseudocorrelation map measures the degree of correlation between neighboring pixels (context) of wrapped phase. The values are normalized in the range [0,1], so that values close to 1 indicate strong reliability (low phase variation) in the neighborhood [22]. Using Euler's formula, (4) can be written in trigonometric form as follows:

$$\chi = \gamma \cos \psi + j\gamma \sin \psi = Re(\chi) + j\, Im(\chi) \quad (5)$$

With: $Re(\chi)$: the real part, $Im(\chi)$: the imaginary part.

The module of χ :

$$\begin{aligned} |\chi| &= \sqrt{\left(Re(\chi)\right)^2 + (Im(\chi))^2} \\ &= \sqrt{(\gamma \cos \psi)^2 + (\gamma \sin \psi)^2} \\ &= |\gamma|\sqrt{(\cos \psi)^2 + (\sin \psi)^2} \end{aligned} \quad (6)$$

For $|\gamma|$ equal to 1, the estimation of the value of coherence loss $|\chi|$ is named pseudocorrelation ρ. Using this value, we go from the space of correlation between the two signals S_A and S_B (coherence) to the space of correlation between values of the interferometric phase:

$$\rho = \frac{\sqrt{(\sum_k \cos(\psi_{x,y}))^2 + (\sum_k \sin(\psi_{x,y}))^2}}{k} \quad (7)$$

With:

x, y: The pixel coordinates.

k : The size of the neighborhood.

Fig. 4a, Fig. 4b, Fig. 4c and Fig. 4d are respectively: the flattened interferogram (corrected from orbital fringes), the coherence, the residues image and the pseudocorrelation image.

We find that the transition zones (zones indicated by arrows) which present discontinuities, are better discriminated in the pseudocorrelation image compared to the coherence image. On the other hand, discontinuities with low contextual correlation values are also localized and described by the image of residues. Thus, the unwrapping guided by pseudocorrelation avoids these areas early in the process to avoid error propagation.

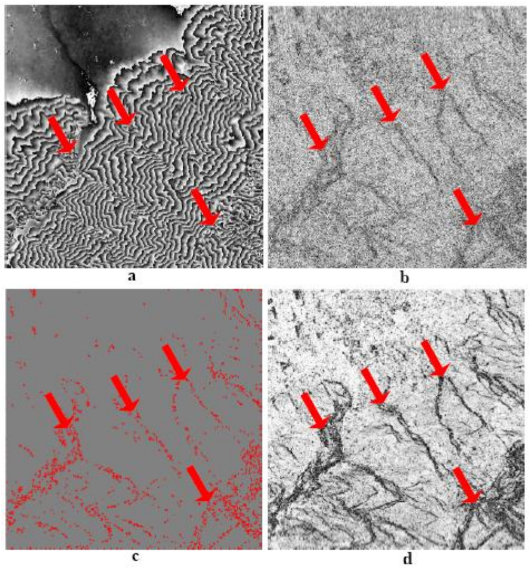


Figure 4. a : interferogram, b : coherence, c : residues image, d : pseudocorrelation image.

B. Unwrapping Path

The unwrapping path depends on the pseudocorrelation value of a set of neighboring pixels.

The proposed algorithm realizes the unwrapping phase as follows:

- The unwrapping process is initialized with pixel value of higher reliability.
- Its vicinity of 8-connectivity is then defined and the neighboring pixels having only a pseudocorrelation value greater than an adaptive threshold value are unwrapped with respect to the central pixel. The threshold is defined relatively to the pseudocorrelation value of central pixel.

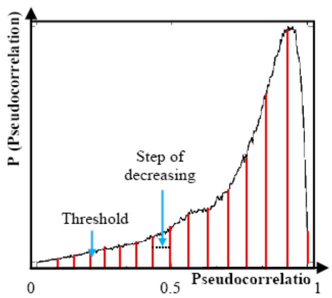


Figure 5. Histogram of pseudocorrelation image.

The threshold follows a decreasing function with respect to the value of the pseudocorrelation where the distance is constant (see Fig. 5).

In the case where the unwrapping phase is carried out on a full-8-connectivity neighborhood with respect to the central pixel with a High quality (reliability), we may unwrap a pixel of low quality before other better pixels.

Nevertheless, we can have four situations in the unwrapping process, regarding the similarity to the pseudocorrelation as:

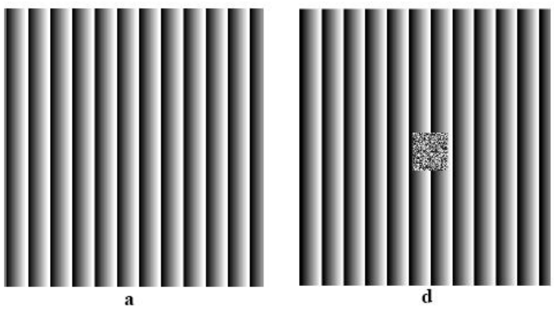
1) All non-unwrapped pixels of the window will be unwrapped with respect to the central pixel and gathered into a new single block of unwrapped pixels.
2) The central pixel of the window (3×3) has been already unwrapped but among its neighbor pixels there are those that were not (do not belonging to any block). These latter will be unwrapped with respect to the central pixel and added to its bl.
3) The central pixel of the window has been already unwrapped, and all its neighbors have been also unwrapped and belong to different blocks. All blocks that are less or equal to the central pixel's block will be unwrapped with respect to the central pixel and merged with its bloc.
4) The central pixel of the window does not belong to any block and among its neighbors there are those that belong to different blocks and others do not belong to any block, the algorithm unwraps the phase as follows:
   - The neighboring pixels which were not already unwrapped will be unwrapped with respect to the central pixel and will be grouped in a new block.
   - The new block needs to be unwrapped with respect to the largest block from the neighboring blocks.

The proposed algorithm has been tested using simulated and real interférogrammes.

V. SIMILATION AND RESULTS

A. Simulation Results

We have generated parallel fringes in the range $[-\pi, \pi]$. Fig. 6a shows a simulated interferogram without discontinuities, Fig. 6b shows the pseudocorrelation image. Its value is constant over the image and is equal to 1. Fig. 6c shows the resulting unwrapped phase.



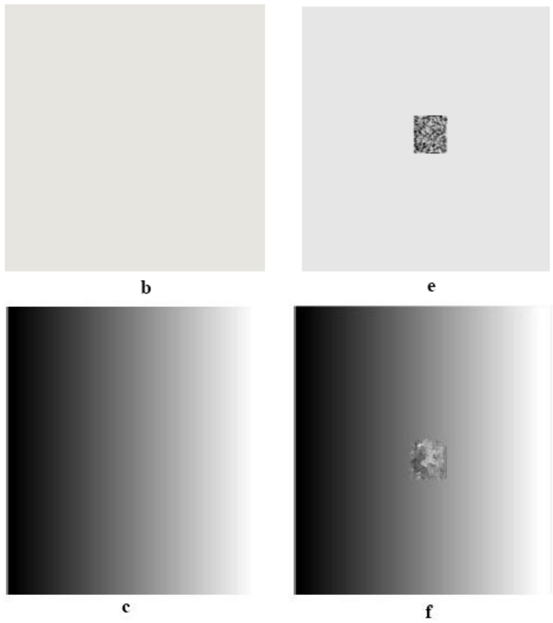


Figure 6. a: Interferogram simulated without noise, b: pseudocorrelation image of a, c: unwrapping result of a. d: Interferogram simulated with noisy area, e:pseudocorrelation image of d, f: unwrapping result of d.

To test the robustness of our algorithm, we have perturbed the simulated interferogram by introducing random noise in small central area (Fig. 6d), Fig. 6e shows its pseudocorrelation image. We note that the unwrapping has been achieved without the propagation of errors that are due to the presence of erroneous phase (see Fig. 6f).

B. Results of Real Interferogram and Comparison

The algorithm was tested on real interferograms. We took the interferogram of Fig. 4a extracted from a pair of tandem sensors ERS1/ERS2 acquired on 3rd and 4th January 1996, for Algiers area (Algeria). This interferogram corresponds to area of hilly terrain (see the fringes pattern in the lower part of Fig. 7a).

Fig. 7b shows the result of phase unwrapping algorithm. The error propagation of phase unwrapping along the path of integration (horizontal direction) is clearly visible. The result of our phase unwrapping algorithm is shown in Fig. 7c. The interferogram is unwrapped entirely (no holes), and the areas of low correlation are unwrapped at the end of the process preventing the error propagation of integration and are kept localized. A three-dimensional representation of the unwrapped phase is shown in Fig. 7d.

The computation time of the proposed algorithm depends on the size of the processed image.

An illustrative example of execution time, whose test conditions are: an image of 500×500 pixels in size, the proposed algorithm executed on a PC system, the execution time to unwrap the image shown in Fig. 7a takes 06 minutes and 42.6 seconds.

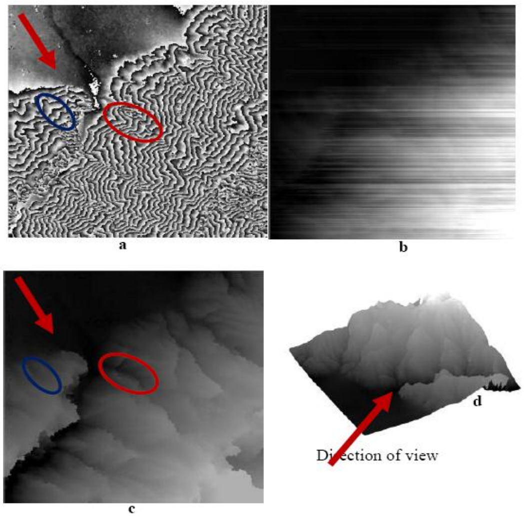


Figure 7. Unwrapping phase of real interferogram. a: real interferogram (phase wrapped), b: unwrapping result of real interferogram using local algorithm, c: unwrapping result of real interferogram using proposed algorithm, d: three dimensional representation of the unwrapped phase.

We can note some particularities of the proposed algorithm, based on the pseudocorrelation quality map:

- The quality map is generated directly from the interferogram and it also models its variability.
- The search for the unwrapping path depends only on the values of the quality matrix, structured as a vector of decreasing values, identified by their position (x, y). This structure minimizes the execution time.
- The sequence with decreasing threshold allows locating and placing the discontinuities at the end of the process avoiding the spread of their errors, and save a continuity of the unwrapped phase at the boundaries of the fringes as shown in the circled areas in Fig. 7a and Fig. 7c.

VI. CONCLUSION

Although the problem of 2D phase unwrapping has been extensively studied and has been the main interest of various studies, particularly in SAR interferometry. The convergence towards an optimal solution reflecting the physical reality is far from being achieved. Different phase unwrapping methods have been developed, some of them are local and they avoid noisy areas, the others are global and seek to minimize the error function holistically. However, each method has its specifications to its results. The proposed algorithm which is based on a contextual pseudocorrelation quality measure uses the notion of 8-incomplete-connectivity and a decreasing threshold to avoid error propagation. After several tests on interferograms of various structures, we found that there is a good continuity of the unwrapped phase at the boundaries of the fringes.

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