Fuzzy Image Enhancement Based on an Adjustable Intensifier Operator

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***Abstract*—Fuzzy image enhancement is an important method in the process of image processing. Fuzzy image enhancement includes steps: gray-level fuzzification, modifying membership using intensifier (INT) operator, and obtaining new gray-levels by defuzzification. This paper proposed an adjustable INT operator with parameter *k*. Firstly, the image's pixels are divided into two regions by the OTSU method (low and high region), and calculate the pixels' membership by fuzzification in each region. Then, the INT operator reduce pixels' membership in the low region and enlarge pixels' membership in the high region. The parameter *k* is determined base on the pixel's location information (neighborhood information), and plays an adjusting role when the INT operator is working. And finally, the result image is obtained by the defuzzification process. In the experiment results, the fuzzy image enhancement with the adjustable intensifier operator achieves a better performance.**

***Keywords*—fuzzy image enhancement, intensifier operator, gradient, threshold**

I. Introduction

Image enhancement is an important branch of image processing. The purpose of image enhancement is to highlight image factors of interest and enhance visual effects. There are two traditional image enhancement methods: (1) Frequency domain method. In this method, the original image is transformed into another domain by a specific filter, then processed on the new domain, and finally, the processed image is inversely transformed back to the original spatial domain to achieve the purpose of enhancement [1−6]. (2) Spatial domain method. This method is to construct the transformation of the pixel gray value, that is, to carry out the enhancement transformation on the image domain, such as the change of gray levels [7−13], histogram correction [14−18]. Fuzzy enhancement method is increasingly applied to image enhancement. Fuzzy enhancement method maps the pixel data of the original image to the fuzzy domain through fuzzification, so that the image to be processed becomes a feature plane image with fuzzy features. Then, the feature plane image information is processed by intensifier (INT) operator, and finally the processed fuzzy set image data is inverse mapped to the spatial domain, so as to obtain the desired effect of the image [19, 20]. The commonly used fuzzification method and INT operator are composed by direct stretching, quadratic function, tangent function, sine function and other nonlinear functions [21−28]. Fuzzy enhancement methods are applied in many fields such as underwater image enhancement, medical image enhancement and remote sensing image enhancement.

Generally, fuzzy image enhancement consists of three steps: gray level fuzzification, suitable operation on membership values and, defuzzification. In this paper, we proposed an adjustable INT operators with a parameter k. The parameter k is related to image pixel's local gradient. The performances of the proposed method is compared with the previous methods [19, 25, 26]. Experimental results show that the enhancement methods with adjustable intensifier operator achieve better image enhancement rather than previous methods.

II. Research Methodology

A. Intensifier (INT) Operator

We note the INT operators in [19, 25, 26] as INT operator 1, INT operator 2, and INT operator 3, respectively. The next work is to introduce INT operator 1, INT operator 2, INT operator 3 and the adjustable INT operator.

The INT operator 1,

$$y = \rho(x) = \begin{cases} 2x^2, & 0 \le x \le 0.5 \\ 1 - 2(1-x)^2, & 0.5 < x \le 1 \end{cases} \quad (1)$$

The INT operator 1 modifies the membership values by increasing the values of x which are above pivotal point $p = 0.5$ and decreasing those which are below it. The disadvantage is that $p \equiv 0.5$ is fixed for different image processing (see Fig. 1).

The INT operator 2,

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$$y = \delta(x) = \begin{cases} \dfrac{x^2}{p}, & 0 \le x \le p \\ 1 - \dfrac{(1-x)^2}{1-p}, & p < x \le 1 \end{cases} \quad (2)$$

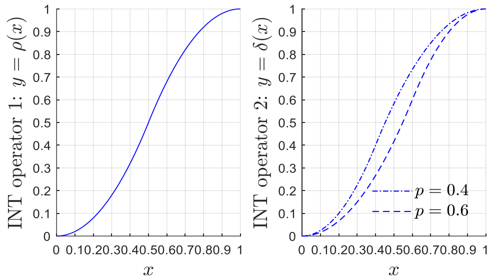

Figure 1. INT operator 1 and INT operator 2.

The INT operator 2 overcomes the shortcomings of the INT operator 1 and can adjust pivotal point p according to different experimental images. Both INT operator 1 and INT operator 2 do not consider the neighborhood information of the test image. This means that when the pixels in image's different positions have the same gray value (membership value), the output value will also be the same after INT operator 1 (or INT operator 2) processing. In the process of image enhancement, we want to take the location information of pixels in the image (pixel's neighborhood) as the reference factor of image enhancement (see Fig. 2).

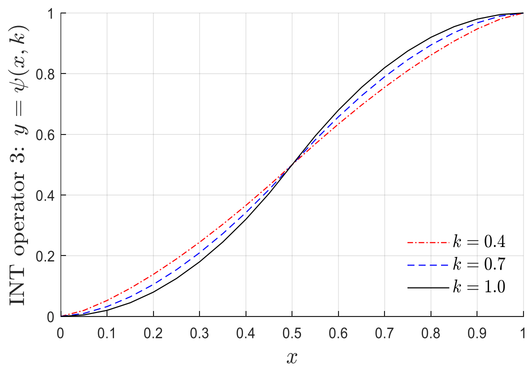

Figure 2. INT operator 3.

The INT operator 3,

$$y = \psi(x,k) = \begin{cases} 2^k x^{k+1}, & 0 \le x \le 0.5 \\ 1 - 2^k (1-x)^{k+1}, & 0.5 < x \le 1 \end{cases} \quad (3)$$

The Eq. (3) was first proposed to be used for image edge detection [27], but our research, we use it as INT operator for image enhancement. In Eq. (3), the k is parameter, and we set $0 \le k \le 1$. In the process of image processing, k plays a role in adjusting the output result according to the position information of pixels (see Fig. 3).

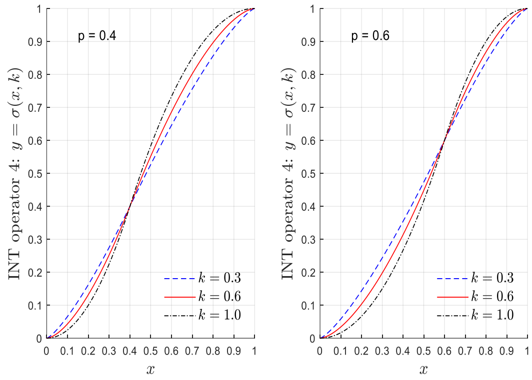

Figure 3. INT operator 4.

Proposition 1. For $0 < x < 0.5$, when k increase, $\psi(x,k)$ decrease, for $0.5 < x < 1$, when k increase, $\psi(x,k)$ increase. When k=1, the $\psi(x,k)$=$\rho(x)$.

Proof. For $0 < x < 0.5$,

$$\frac{\partial \psi(x,k)}{\partial k} = 2^k (\ln 2) x^{k+1} + x^{k+1} (\ln x) 2^k$$

$$= 2^k x^{k+1} \ln(2x) < 0,$$

for $0.5 < x < 1$,

$$\frac{\partial \psi(x,k)}{\partial k} = -2^k (\ln 2)(1-x)^{k+1} - (1-x)^{k+1} [\ln(1-x)] 2^k$$

$$= -2^k x^{k+1} \ln[2(1-x)] > 0.$$

Obviously, $\psi(x,1)$=$\rho(x)$. Proposition true.

The INT operator 4 (the adjustable INT operator),

$$y = \sigma(x,k) = \begin{cases} \dfrac{x^{k+1}}{p^k}, & 0 \le x \le p \\ 1 - \dfrac{(1-x)^{k+1}}{(1-p)^k}, & p < x \le 1 \end{cases} \quad (4)$$

The INT operator 4 has an adjustable pivotal point p and the position factors of pixels in the image (pixel neighborhood information) are associated by adjusting the parameter k.

Proposition 2. For $0 < x < p$, when k increase, $\sigma(x,k)$ decrease, for $p < x < 1$, when k increase, $\sigma(x,k)$ increase. When k=1, the $\sigma(x,k)$=$\delta(x)$.

Proof. For $0 < x < p$,

$$\frac{\partial \sigma(x,k)}{\partial k} = \frac{x^{k+1} (\ln x) p^k - p^k (\ln p) x^{k+1}}{(p^k)^2}$$

$$= \frac{p^k x^{k+1} \ln(x/p)}{p^{2k}} < 0,$$

for $p < x < 1$,

$$\frac{\partial\sigma(x,k)}{\partial k}=-\frac{(1-x)^{k+1}[\ln(1-x)](1-p)^k-(1-p)^k[\ln(1-p)](1-x)^{k+1}}{[(1-p)^k]^2}$$

$$=-\frac{(1-p)^k(1-x)^{k+1}}{(1-p)^{2k}}\ln(\frac{1-x}{1-p})>0\,.$$

Obviously, $\sigma(x,k)=\delta(x)$. Proposition true.

Proposition 1 and 2 show that the values of functions and can be regularly changed by adjusting the parameter k. For $\forall x\in(0,0.5)\cup(0.5,1)$, The value of $y=\psi(x,k)$ function keeps increasing or decreasing. For $\forall x\in(0,p)\cup(p,1)$, The value of $y=\sigma(x,k)$ function also keeps increasing or decreasing. This means that the mapping k to $y=\psi(x,k)$ ($y=\sigma(x,k)$) is bijection. We can adjust parameter k to get a unique value of the $y=\psi(x,k)$ ($y=\sigma(x,k)$) function.

B. Fuzzy Contrast Enhancement

Image $\mathrm{I}=\{x_{ij}\mid i=1,2,3,\ldots,m,\ j=1,2,3,\ldots,n\}$, where x_{ij} is the gray scale of the image's pixel in row i and column j. Fuzzy contrast enhancement includes three steps as follow:

1) Gray-level fuzzification

$$\mu_{ij}=\mu(x_{ij})=\frac{x_{ij}-x_{\min}}{x_{\max}-x_{\min}} \tag{5}$$

where, $x_{\max}$ ($x_{\min}$) is the maximum(minimum) pixel's gray scale in the original image.

2) Membership modification using INT operator

INT operator 1: $\hat{y}_{ij}=\rho(\mu_{ij})$,

INT operator 2: $\hat{y}_{ij}=\delta(\mu_{ij})$,

INT operator 3: $\hat{y}_{ij}=\psi(\mu_{ij},k_{ij})$,

INT operator 4: $\hat{y}_{ij}=\sigma(\mu_{ij},k_{ij})$.

In Eq. (2) and Eq. (4), $p=\mu(T)$, T is the original image threshold by the OTSU method.

3) New gray-levels by defuzzification

$$y_{ij}=\hat{y}_{ij}(x_{\max}-x_{\min})+x_{\min} \tag{6}$$

$\mathrm{I}_{enh}=\{y_{ij}\mid i=1,2,3,\ldots,m,\ j=1,2,3,\ldots,n\}$ is the result image.

C. Image Pixel's Local Gradient

$\mathrm{I}_G=\{g_{ij}\mid i=1,2,3,\ldots,m,\ j=1,2,3,\ldots,n\}$ is the local gradient matrix of image I.

$$g_{ij}=\begin{cases}\sqrt{(x_{i+1j}-x_{ij})^2+(x_{ij+1}-x_{ij})^2}, & i=1,2,\ldots,m-1,\ j=1,2,\ldots,n-1,\\ 0, & others.\end{cases} \tag{7}$$

I_G shows the changing trend of all pixels' gray scale of image I.

III. Experimental Results

For image enhancement effect evaluation, commonly used algorithms include Mean Squared Error (MSE), Peak Signal-Noise Ratio (PSNR) and Structural Similarity (SSIM). The smaller MSE or the higher PSNR (SSIM) indicates better enhancement effect. In this paper, The MSE, PSNR, and SSIM value are used to evaluate the effect of the enhancement methods.

$$MSE=\frac{1}{M\times N}\sum_{i=1}^{M}\sum_{j=1}^{N}(x_{i,j}-y_{i,j})^2 \tag{8}$$

$$PSNR=10\times\log_{10}\frac{(2^n-1)^2}{MSE}dB \tag{9}$$

$$SSIM(x,y)=\frac{(2\mu_x\mu_y+c_1)(2\sigma_{xy}+c_2)}{(\mu_x^2+\mu_y^2+c_1)(\sigma_x^2+\sigma_y^2+c_2)} \tag{10}$$

In Eq. (9), n = 8. In Eq. (10), μ_x is the mean of x, μ_y is the mean of y, σ_x^2 is the variance of x, σ_y^2 is the variance of y, σ_{xy} is the covariance of x and y. c_1 and c_2 are constants [28]. For experimentation, we considered image lena and 22 test images from Miscellaneous (MISC) dataset (http://sipi.usc.edu/database/database.php?volume=misc).

In this section, the parameter x (x_{ij}) in the Eq. (3) and Eq. (4) are calculated as follow:

$$k=k_{ij}=\frac{a\times g_{ij}}{g_{\max}-g_{\min}} \tag{11}$$

where, $g_{\max}$ ($g_{\min}$) is the maximum(minimum) value in I_G, $0\le a\le 1$.

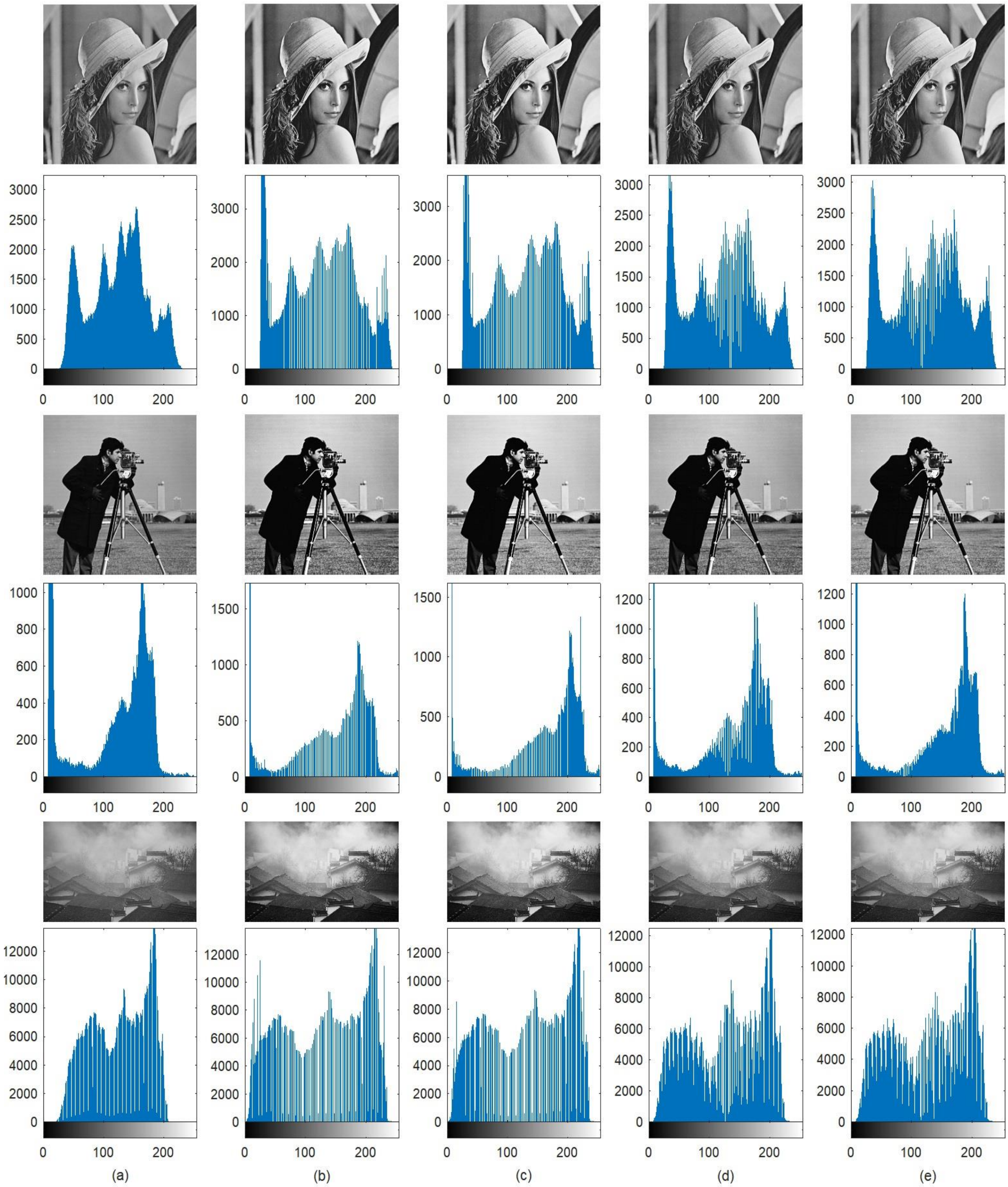

Figure 4. Comparison of histogram of original and enhanced images. (a) Original image, (b) processed by INT operator 1, (c) processed by INT operator 2, (d) processed by INT operator 3 with $a = 0.5$, (e) processed by INT operator 4 with $a = 0.5$.

In Fig. 4, compared with Fig. 4(b), the histogram in Fig. 4(d) is more uniform, Fig. 4(d) is more visually natural. Fig. 4(e) has a similar effect compared to Fig. 4(c). The result image processed by the adjustable INT operator is more visually similar to the original image.

TABLE I. MSE TEST RESULTS

Test image	Processed by enhancement method with INT operator 1, 2, 3, and 4.							
			a = 0.4		a = 0.6		a = 0.8	
	operator 1	operator 2	operator 3	operator 4	operator 3	operator 4	operator 3	operator 4
APC	297.54	553.23	131.47	244.42	67.68	126.54	23.29	42.73
Aerial	472.05	612.48	234.06	290.85	135.14	161.60	58.16	64.49
Aerial2	683.01	502.06	345.30	**249.17**	193.69	**139.93**	74.47	**55.19**
Airplane (U-2)	260.34	192.68	152.95	**102.69**	93.52	**60.62**	39.03	**24.63**
Airplane	404.62	217.25	214.59	**104.22**	122.77	**59.85**	45.32	**25.35**
Airplane2	864.83	747.31	390.93	**337.21**	199.60	**170.57**	60.97	**52.09**
Airport	629.68	435.15	314.23	**209.42**	177.67	**118.69**	71.66	**49.86**
Car and APCs	304.26	567.22	134.36	249.96	70.06	128.73	24.38	43.00
Car and APCs2	261.88	309.50	123.68	146.25	69.09	81.21	28.33	32.80
Chemical plant	476.60	451.87	248.49	**227.58**	149.38	**135.01**	69.66	**63.70**
Clock	352.62	336.53	194.31	**183.28**	117.06	**109.78**	48.66	**46.92**
Couple	393.42	493.30	176.23	218.36	91.40	111.59	29.98	35.75
Fishing Boat	455.39	834.87	213.60	383.01	115.73	202.84	42.19	70.54
Male	433.67	671.62	207.07	306.97	111.64	160.46	38.66	53.25
Moon surface	292.39	314.51	134.49	144.06	73.95	78.88	29.93	31.58
Stream and bridge	588.69	585.35	302.07	**300.34**	179.10	**178.35**	81.15	**80.82**
Tank	375.49	396.68	174.29	180.01	94.17	97.28	35.64	36.62
Tank2	245.98	286.85	111.76	126.47	60.59	67.03	23.60	24.59
Tank3	414.57	500.41	196.48	230.95	108.55	124.85	42.94	47.06
Truck and APCs	419.74	407.91	196.41	**184.70**	106.53	**97.86**	38.84	**34.71**
Truck and APCs2	529.36	402.96	263.95	**186.55**	151.48	**102.89**	63.37	**41.09**
Truck	313.34	418.47	147.94	179.07	81.90	91.79	32.14	**31.37**

TABLE II. PSNR TEST RESULTS

Test image	Processed by enhancement method with INT operator 1, 2, 3, and 4.							
			a = 0.4		a = 0.6		a = 0.8	
	operator 1	operator 2	operator 3	operator 4	operator 3	operator 4	operator 3	operator 4
APC	23.3954	20.7017	26.9426	24.2495	29.8259	27.1084	34.4588	31.8239
Aerial	21.3909	20.2599	24.4375	23.4942	26.8228	26.0464	30.4845	30.0358
Aerial2	19.7865	21.1232	22.7488	**24.1659**	25.2597	**26.6716**	29.4113	**30.7126**
Airplane (U-2)	23.9754	25.2824	26.2853	**28.0156**	28.4218	**30.3045**	32.2170	**34.2158**
Airplane	22.0604	24.7611	24.8148	**27.9511**	27.2400	**30.3598**	31.5680	**34.0917**
Airplane2	18.7615	19.3958	22.2098	**22.8518**	25.1292	**25.8118**	30.2794	**30.9635**
Airport	20.1396	21.7444	23.1583	**24.9207**	25.6348	**27.3867**	29.5780	**31.1529**
Car and APCs	23.2983	20.5933	26.8480	24.1520	29.6762	27.0340	34.2602	31.7960
Car and APCs2	23.9498	23.2242	27.2079	26.4798	29.7369	29.0349	33.6084	32.9719
Chemical plant	21.3493	21.5807	24.1778	**24.5595**	26.3877	**26.8271**	29.7013	**30.0892**
Clock	22.6577	22.8606	25.2458	**25.4997**	27.4467	**27.7257**	31.2592	**31.4170**
Couple	22.1822	21.1997	25.6701	24.7391	28.5216	27.6545	33.3628	32.5976
Fishing Boat	21.5470	18.9146	24.8348	22.2987	27.4962	25.0593	31.8788	29.6465
Male	21.7592	19.8596	24.9697	23.2598	27.6526	26.0770	32.2582	30.8676
Moon surface	23.4712	23.1545	26.8441	26.5455	29.4414	29.1611	33.3695	33.1361
Stream and bridge	20.4320	20.4566	23.3297	**23.3546**	25.5998	**25.6181**	29.0381	**29.0558**
Tank	22.3849	22.1464	25.7181	25.5778	28.3916	28.2505	32.6112	32.4932
Tank2	24.2218	23.5542	27.6478	27.1108	30.3065	29.8678	34.4023	34.2241
Tank3	21.9549	21.1376	25.1976	24.4957	27.7744	27.1670	31.8025	31.4039
Truck and APCs	21.9010	22.0252	25.1991	**25.4662**	27.8562	**28.2246**	32.2378	**32.7258**
Truck and APCs2	20.8933	22.0781	23.9156	**25.4228**	26.3273	**28.0070**	30.1116	**31.9939**
Truck	23.1706	21.9141	26.4300	25.6005	28.9979	28.5030	33.0610	**33.1663**

Tables I−III are the MSE, PSNR, and SSIM values between 22 processed images and original images, respectively. Table I shows that operator 4 (a = 0.4, 0.6, 0.8) has a lower MSE value compared with operator 1 and operator 2. Operator 4 compared to operator 3, when a = 0.4, 0.6, and 0.8, there are 10, 10 and 11 smaller values of MSE, respectively. Tables II and III show that operator 4 (a= 0.4, 0.6, 0.8) has a higher PSNR value (SSIM value) compared with operator 1 and operator 2. Operator 4 compared to operator 3, when a = 0.4, 0.6, and 0.8, there are 10, 10 and 11 higher values of PSNR (SSIM), respectively.

On the other hand, As the a value increases, the MSE value in column operator 3, and operator 4 decreases, respectively. And when the a value increases, the SSIM (PSNR) value in column operator 3, and operator 4 increases, respectively. This indicates that more appropriate result images can be obtained by adjusting the value of a. That's one of the advantages of adjustable INT operators [29].

TABLE III. SSIM TEST RESULTS

Test image	Processed by enhancement method with INT operator 1, 2, 3, and 4.							
			a = 0.4		a = 0.6		a = 0.8	
	operator 1	operator 2	operator 3	operator 4	operator 3	operator 4	operator 3	operator 4
APC	0.9235	0.9382	0.9611	**0.9685**	0.9782	**0.9822**	0.9912	**0.9927**
Aerial	0.9080	0.8945	0.9497	0.9436	0.9691	0.9658	0.9853	0.9841
Aerial2	0.9512	0.9403	0.9733	0.9669	0.9839	0.9798	0.9926	0.9908
Airplane (U-2)	0.6765	0.7868	0.8179	**0.8929**	0.8952	**0.9410**	0.9607	**0.9788**
Airplane	0.9690	0.9678	0.9830	**0.9832**	0.9894	**0.9895**	0.9942	0.9941
Airplane2	0.9720	0.9713	0.9855	0.9851	0.9919	0.9915	0.9967	0.9966
Airport	0.8549	0.8856	0.9285	0.9416	0.9601	0.9657	0.9842	0.9852
Car and APCs	0.8802	0.9015	0.9384	**0.9485**	0.9651	**0.9708**	0.9868	**0.9889**
Car and APCs2	0.9005	0.9061	0.9455	**0.9487**	0.9669	**0.9688**	0.9848	**0.9857**
Chemical plant	0.8750	0.8900	0.9322	**0.9386**	0.9578	**0.9610**	0.9780	**0.9793**
Clock	0.9617	0.9594	0.9787	0.9776	0.9863	0.9858	0.9924	0.9918
Couple	0.8898	0.9023	0.9460	**0.9509**	0.9706	**0.9730**	0.9897	**0.9905**
Fishing Boat	0.8788	0.9016	0.9365	**0.9483**	0.9642	**0.9706**	0.9866	**0.9887**
Male	0.8020	0.8537	0.8848	**0.9184**	0.9323	**0.9527**	0.9753	**0.9830**
Moon surface	0.8854	0.8878	0.9373	**0.9386**	0.9610	**0.9617**	0.9802	**0.9805**
Stream and bridge	0.8759	0.8765	0.9346	**0.9349**	0.9610	**0.9611**	0.9822	0.9822
Tank	0.9184	0.9187	0.9560	**0.9570**	0.9740	**0.9744**	0.9885	**0.9887**
Tank2	0.8749	0.8773	0.9337	**0.9350**	0.9613	**0.9620**	0.9838	**0.9845**
Tank3	0.8978	0.9099	0.9464	**0.9522**	0.9687	**0.9719**	0.9867	**0.9880**
Truck and APCs	0.8770	0.8797	0.9360	**0.9370**	0.9633	**0.9639**	0.9857	**0.9861**
Truck and APCs2	0.8527	0.8642	0.9212	**0.9268**	0.9533	**0.9565**	0.9797	**0.9814**
Truck	0.8920	0.8914	0.9444	0.9441	0.9674	**0.9681**	0.9860	**0.9875**

IV. CONCLUSION

This paper proposed an adjustable intensifier (INT) operator. It can reduce pixels' membership which is lower than pivotal point p, and enlarge pixels' membership which is higher than pivotal point p. That means that the methods can reduce and enlarge pixels' gray-level in the low and high regions, respectively, and can enhance image contrast. Through the function of parameter k in the image enhancement process, the methods can also make proper use of the image pixels' neighborhood information. More result images can be obtained by adjusting a.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

AUTHOR CONTRIBUTIONS

The author Libao Yang conducted the research; the authors Suzelawati Zenian and Rozaimi Zakaria analyzed the data; Libao Yang wrote the paper; all the authors have the same contribution, and had approved the final version.

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